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Multirate partial differential equations for the solution of field-circuit coupled problems

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Abstract

Multirate partial differential equations (MPDEs) are a relatively new concept to deal with multirate phenomena. This abstract heads for the simulation of a low-frequency energy application with pulsed excitation, namely a buck converter, using MPDEs. A linear field-circuit coupled problem described in a single equation system is considered. The differential algebraic equations are rewritten as MPDEs and efficiently solved by a Galerkin ansatz and time discretization.

Key words: Multirate partial differential equations, Energy applications

1 Multirate formulation and solution

Consider the example of a simplified buck converter as in [2]. Its solution consists of fast periodically varying ripples and a slowly varying envelope as depicted in Fig. 1. This makes conventional time discretization inefficient as many steps are necessary to properly resolve the solution. The model of the buck converter consists of a circuit part and a finite element model of the coil which are strongly coupled, i.e., described in a single system of differential algebraic equations (DAEs) with dimension 15791 [3]. The DAEs with matrices $\mathbf{A}$ and $\mathbf{B}$ are equivalently rewritten as multirate partial differential equations (MPDEs) [1] by splitting the time into two time scales of different rate t_1 and t_2

$$\mathbf{A}\left(\frac{\partial\widehat{\mathbf{x}}}{\partial t_1}+\frac{\partial\widehat{\mathbf{x}}}{\partial t_2}\right)+\mathbf{B}\,\widehat{\mathbf{x}}(t_1,t_2)=\widehat{\mathbf{c}}(t_1,t_2)\,. \tag{1}$$

If $\widehat{\mathbf{c}}(t,t)=\mathbf{c}(t)$ is satisfied, the solution of the DAEs and MPDEs relate by $\mathbf{x}(t)=\widehat{\mathbf{x}}(t,t)$, where $\mathbf{c}(t)$ and $\mathbf{x}(t)$ are the excitation and solution of the DAEs, respectively. To solve the MPDEs, the solution is expanded into periodic basis functions $p_k(\tau)$ and coefficients $w_{j,k}(t_1)$

$$\widehat{x}_j(t_1,t_2)=\sum_{k=0}^{N_\mathrm{p}}p_k(\tau)w_{j,k}(t_1)\ \text{with}\ \tau=\frac{t_2}{T_\mathrm{s}}\ \mathrm{mod}\ 1. \tag{2}$$

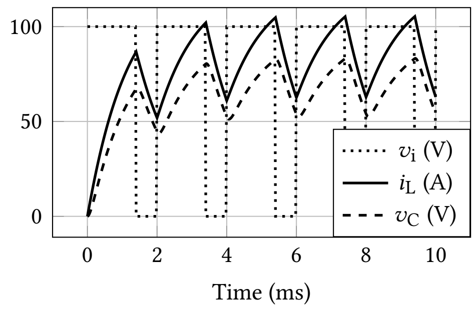

Figure 1: Solution of the simplified buck converter at $f_\mathrm{s} = \frac{1}{T_\mathrm{s}} = 500\,\mathrm{Hz}$.

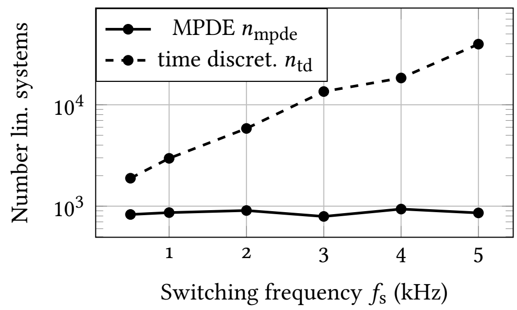

Figure 2: Comparison of number of solved equation systems versus switching frequency.

The time scale t_2 is associated with the fast periodically varying ripples, the time scale t_1 with the slowly varying envelope. Applying a Galerkin ansatz with respect to t_2, the MPDEs (1) reduce to DAEs in t_1 whose unknowns are the coefficients $w_{j,k}(t_1)$. These DAEs exhibit a much slower dynamic than the original ones as the fast periodically varying ripples are taken into account by the Galerkin ansatz. Therefore less time steps are needed for the solution. A drawback is the larger equation systems. As basis $p_k(\tau)$, the problem specific pulse width modulation (PWM) basis functions as introduced by Gyselinck et al. [2] are used.

2 Numerical results

The relative discrete ℓ^2-error of the MPDE solution towards a reference solution is calculated. The reference solution is obtained by conventional time discretization with fine rel./abs. tolerance of 10^{-6}. The simulation time interval is fixed to $t \in [0, 10]$ ms. For the MPDE approach, the number of basis functions and rel./abs. tolerance for time discretization are $N_\mathrm{p} = 2$ and $tol = 10^{-2}$, respectively. Conventional time discretization (backward euler) is applied to the original DAE such that the same ℓ^2-error compared to the reference solution is obtained. The number of solved linear equation systems (n_td for conventional time discretization, n_mpde for MPDE approach) are compared, see Fig. 2. Increasing f_s, more ripples have to be resolved which leads to higher n_td in conventional time discretization while n_mpde stays almost constant. However, the actual efficiency of the MPDE approach depends on the efficiency of the linear solver and the switching frequency f_s.

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